Nuclear thermal to electric power conversion carries the promise of longer duration missions and higher scienti_c data transmission rates back to Earth for a range of missions, including both Mars rovers and deep space missions. A free-piston Stirling convertor is a candidate technology that is considered an e_cient and reliable power conversion device for such purposes. While already very e_cient, it is believed that better Stirling engines can be developed if the losses inherent in current designs could be better understood. However, they are di_cult to instrument and so e_orts are underway to simulate a complete Stirling engine numerically. This has only recently been attempted and a review of the methods leading up to and including such computational analysis is presented. And _nally it is proposed that the quality and depth of Stirling loss understanding may be improved by utilizing the higher _delity and e_ciency of recently developed numerical methods. One such method, the Ultra HI-FI technique is presented in detail.

How to Overcome Numerical Challenges to Modeling Stirling Engines

By

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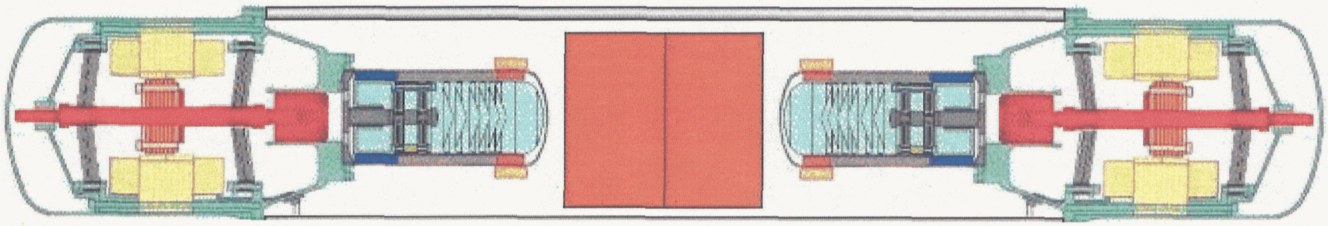
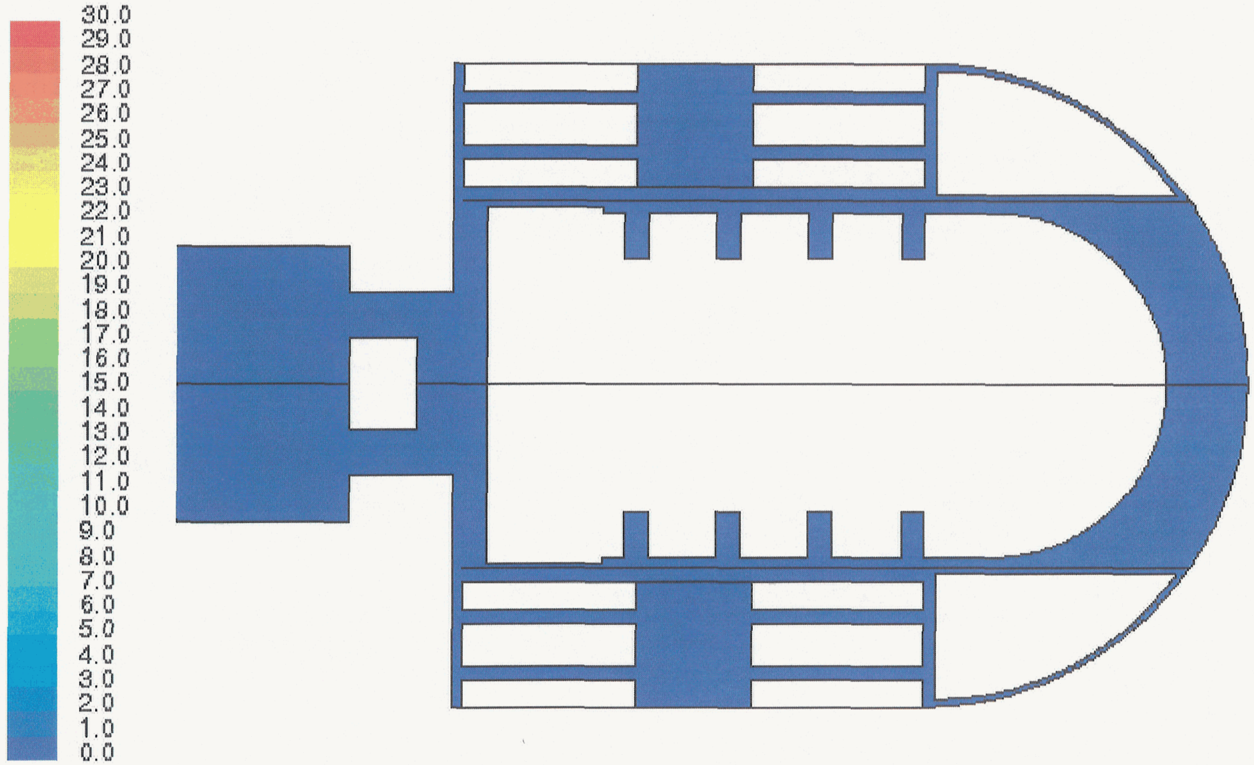
Stirling Analysis and Modeling I
Session EC-8, 10:00 A.M., August 17, 2004
2nd International Energy Conversion Engineering Conference
Providence, RI

Stirling Convertor CFD

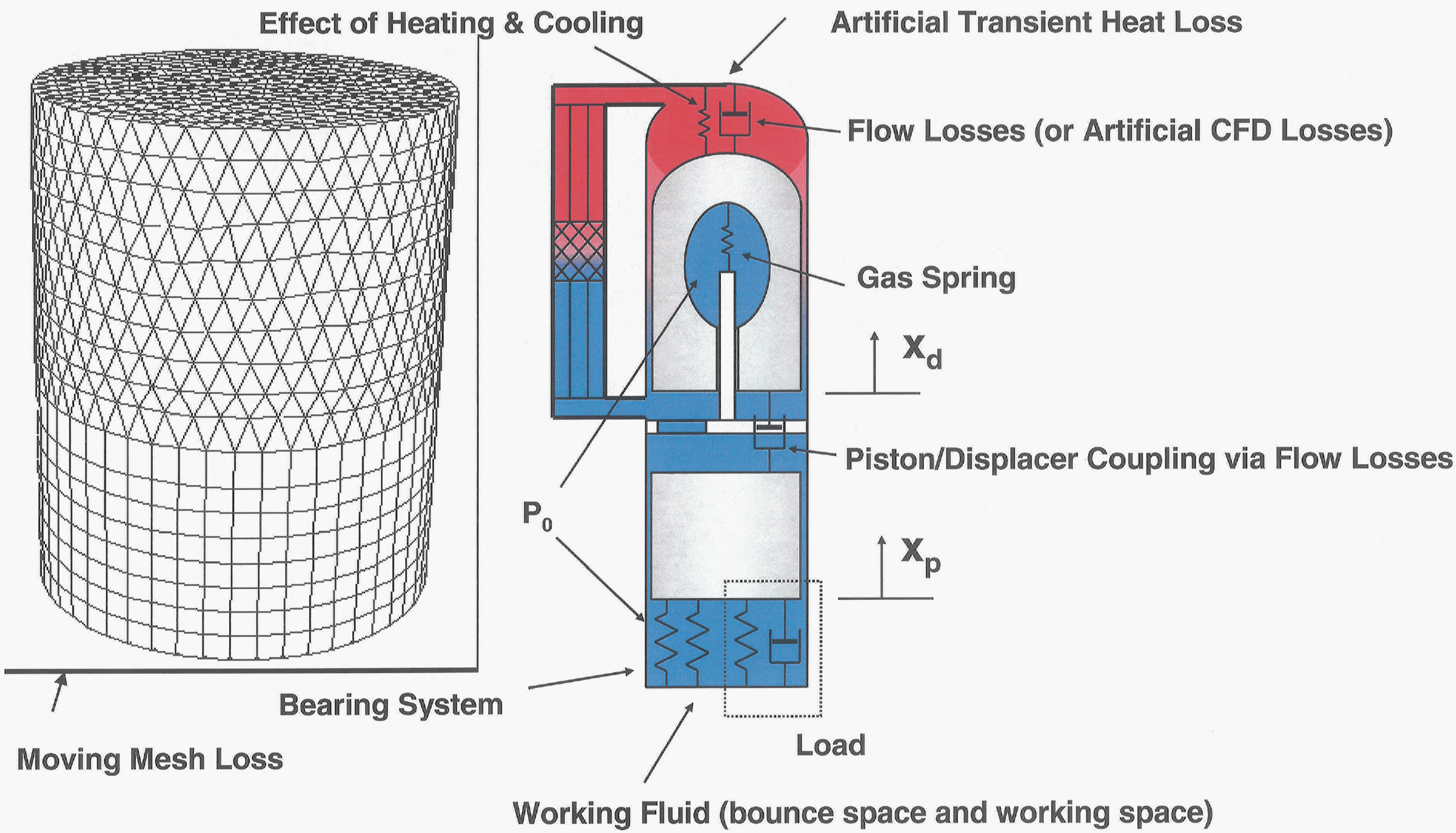
System Overview





Contours of Velocity Magnitude (m/s) (Time=0.0000e+00) Mar 22, 2004
FLUENT 6.1 (axi, dp, segregated, dynamesh, ske, unsteady)

Schematic with Springs and Dampers



Martini Nomenclature (West)
(See also Organ and Urieli)

1. First Order (ideal loss-free analysis, use correction factor)
   - Schmidt or Cooke-Yarborough
2. Second Order (decoupled losses, use adiabatic analysis)
   - Berchowitz, D.M. (includes dynamics)
     - Subtract heat losses
       - Appendix gap (shuttle and pumping)
       - Regenerator imperfections
     - Subtract power losses
       - Pressure drop (non-uniform flow across regenerators)
       - Transient heat transfer
3. Third Order (control volumes or nodes, direct solve 1D equations)
   - Finkelstein (first), Urieli, Berchowitz (includes dynamics)
   - Implicit Space and Time (GLIMPS, SAGE, finite difference, 1D)
   - Linearized harmonic analysis (HFAST, finite volume, 1D)

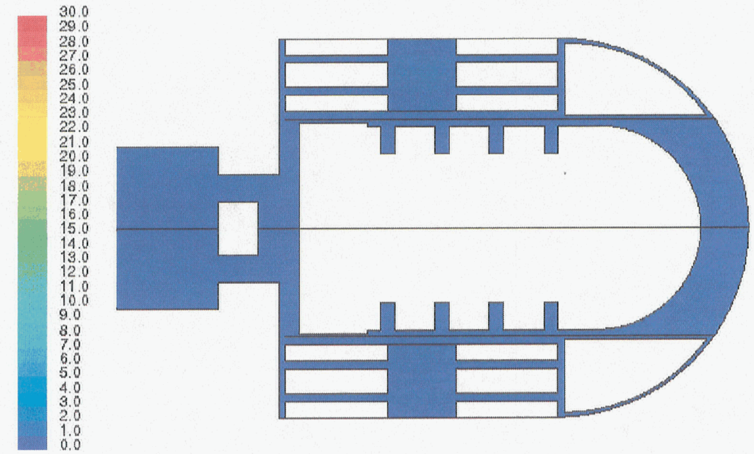
Fourth Order? Full CFD Analysis (2D or 3D)

Flow distortion (oscillating flow distorts velocity and temperature profiles) in the regenerator has significant effects on performance and must be modeled with at least two dimensions (Berchowitz).

- Manifest (2D, D. Gedeon)
  - Beam and Warming
  - Uses thermal non-equilibrium porous media model
- Fluent (2D and 3D)
  - SIMPLER and PISO
  - Uses thermal equilibrium porous media models
- Modified CAST (Tew, Ibrahim, Peric, et. al.)
  - SIMPLE
- CFD-ACE (2D and 3D)
  - SIMPLE & SIMPLER
  - Uses thermal equilibirium porous media models
- Star-HPC, CFX
  - SIMPLE & PISO

Whole Engine Simulation

- Mahkamov, et. al, University of Durham
- Ibrahim, et. al., Cleveland State University
- Wilson, et. al., NASA GRC
- Recent proprietary commercial attempts



Contours of Velocity Magnitude (m/s) (Time=0.0000e+00) Mar 22, 2004
FLUENT 6.1 (axi, dp, segregated, dynamesh, ske, unsteady)

ALL USE 2nd ORDER SIMPLE/PISO RANS APPROACH

TOO SLOW!

Too Much Numerical Error

$$\frac{\Delta F}{\Delta t} = \frac{F(t+\Delta t) - F(t)}{\Delta t} \approx \frac{dF(t)}{dt}$$

F(t)=Exp[i wt]

$$\frac{\Delta F}{\Delta t} = \frac{e^{i\omega t}(e^{i\omega\Delta t} - 1)}{\Delta t} = i\omega e^{i\omega t}\left(\frac{e^{i\omega\frac{\Delta t}{2}} - e^{-i\omega\frac{\Delta t}{2}}}{2i}\right)\left(\frac{e^{i\omega\frac{\Delta t}{2}}}{\omega\frac{\Delta t}{2}}\right)$$

$$= i\omega e^{i\omega t}\left(\frac{\sin\left(\frac{\omega\Delta t}{2}\right)}{\frac{\omega\Delta t}{2}}\right)e^{i\omega\frac{\Delta t}{2}} = \frac{dF}{dt}\left(\frac{\sin\left(\frac{\omega\Delta t}{2}\right)}{\frac{\omega\Delta t}{2}}\right)e^{i\omega\frac{\Delta t}{2}}$$

Dispersion

Dissipation

Turbulence Modeling for Oscillating Flows

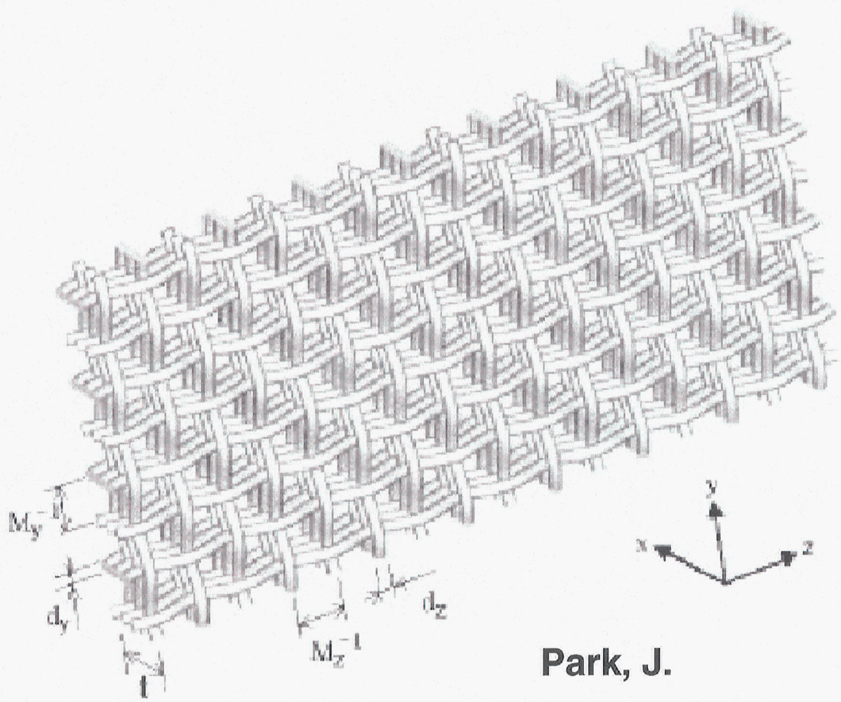
Incorrect Pressure Drop and Heat Transfer

- Wavelet Analysis of Multiscales
- DNS with Wall Layer Approximations
- Detached Eddy Simulation (RANS/LES)
- Cascade Technologies, V2F for low Reynolds number heat transfer
- Ultra-wave Kolmogorov Scale Resolution Techniques

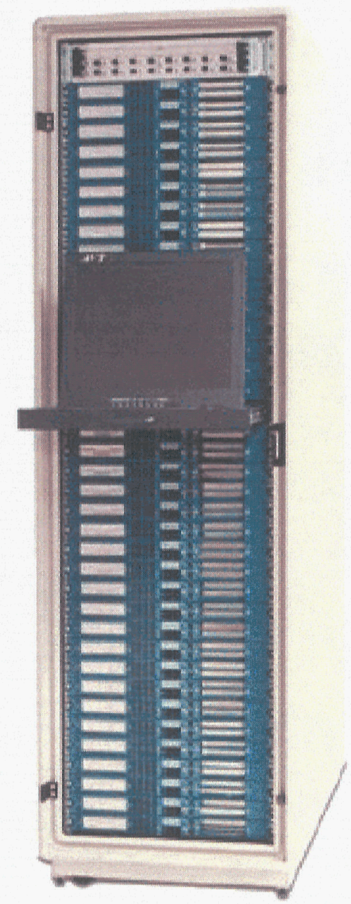
Full DNS Jet Simulations with Re>100,000 are now possible

Parallel Solution

- Assume 20 micron wire diameter, 25mm by 5mm by 5mm regenerator region to simulate
- Might have (at most) 312000 wires in 2D or 8 million in 3D depending on porosity
- Regularized screen meshes may enable more detailed analysis



Park, J.

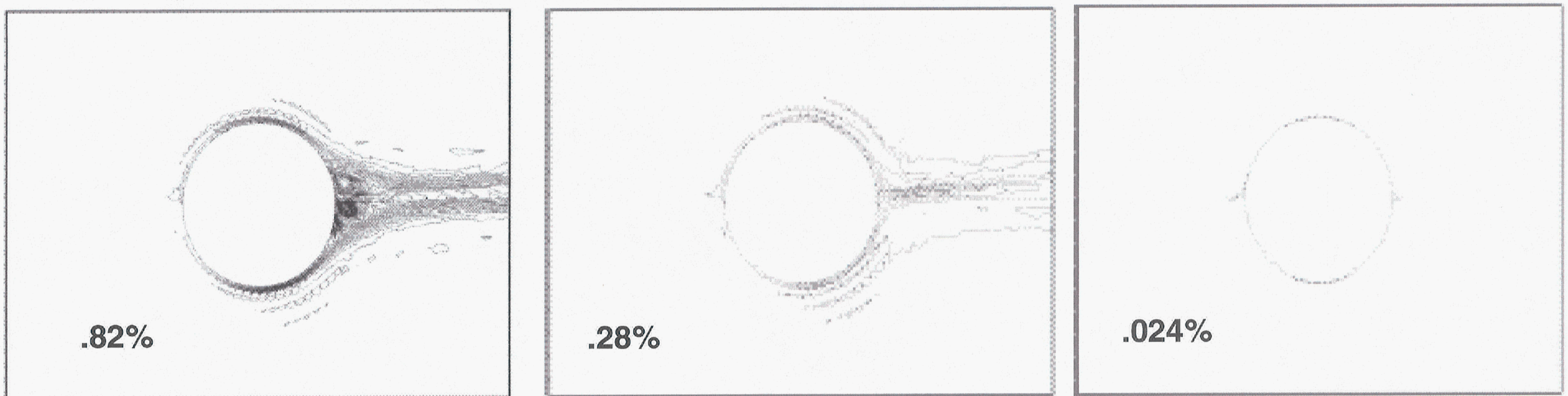


120 Million Cells, 128 processors, 24 hours

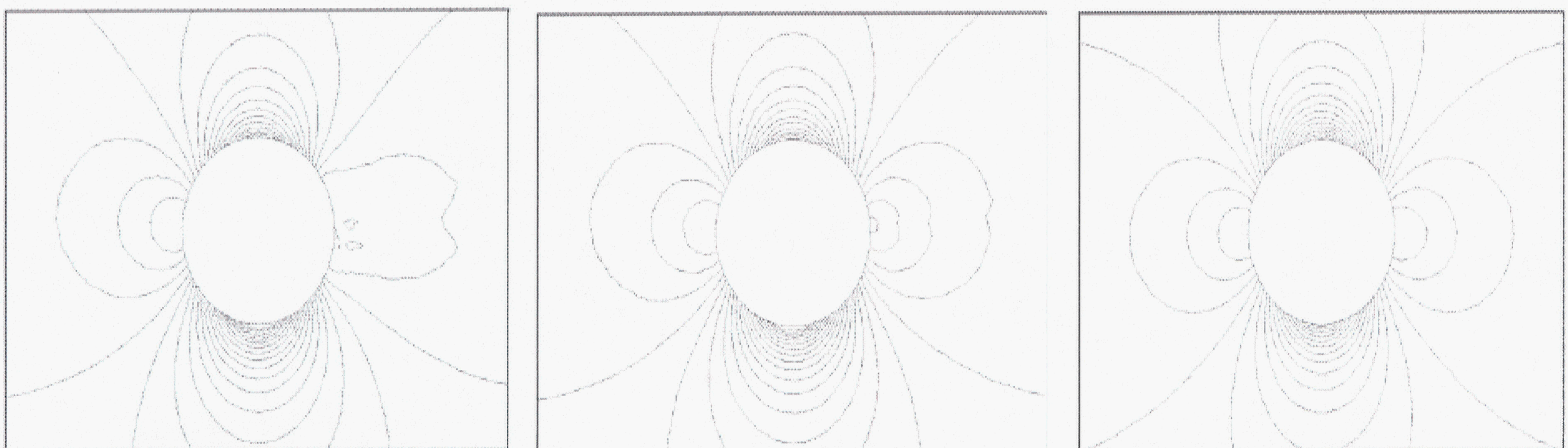
STAR-HPC

Curved Spectral Volumes (Z.J. Wang)
Entropy Production and Density Contour
M=0.3, Inviscid Flow

2nd Order Piecewise Linear | 2nd Order "High Fidelity Grid" | 3rd Order Piecewise Quadratic

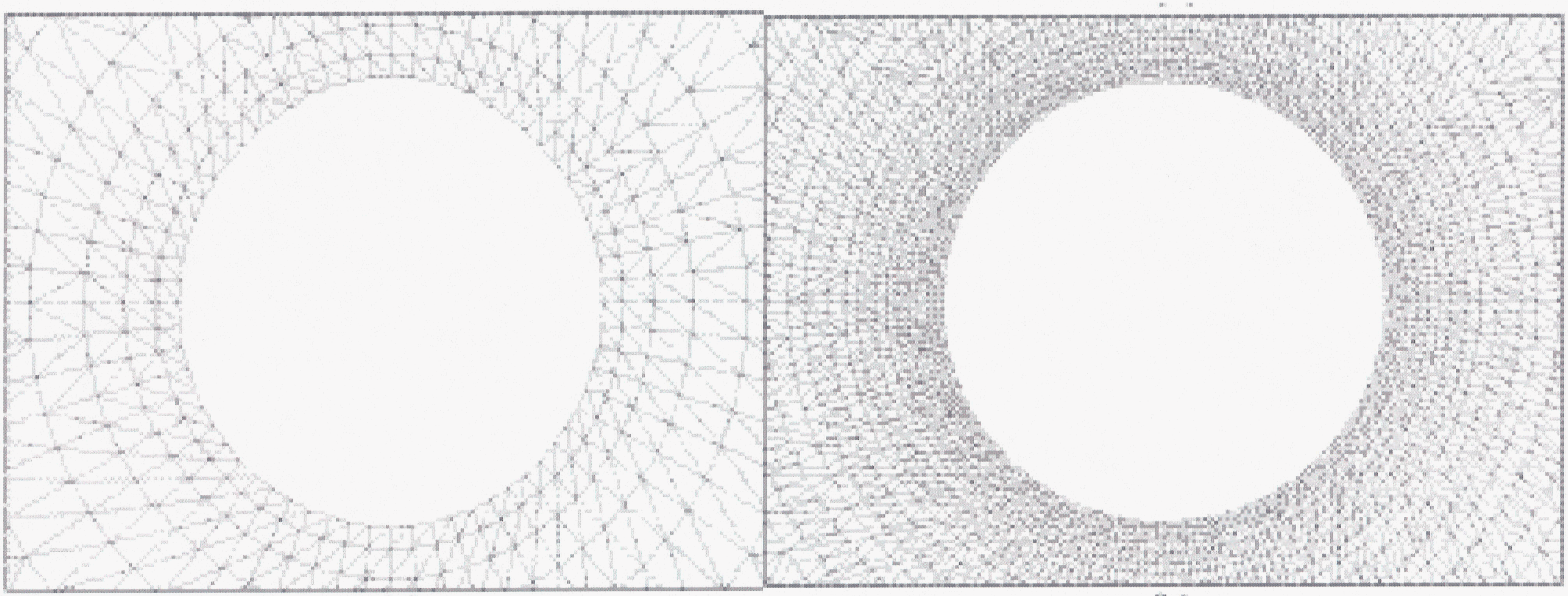


(a) (b) (c)



(a) (b) (c)

Curved Spectral Volumes (Z.J. Wang)
Same Grid Resources

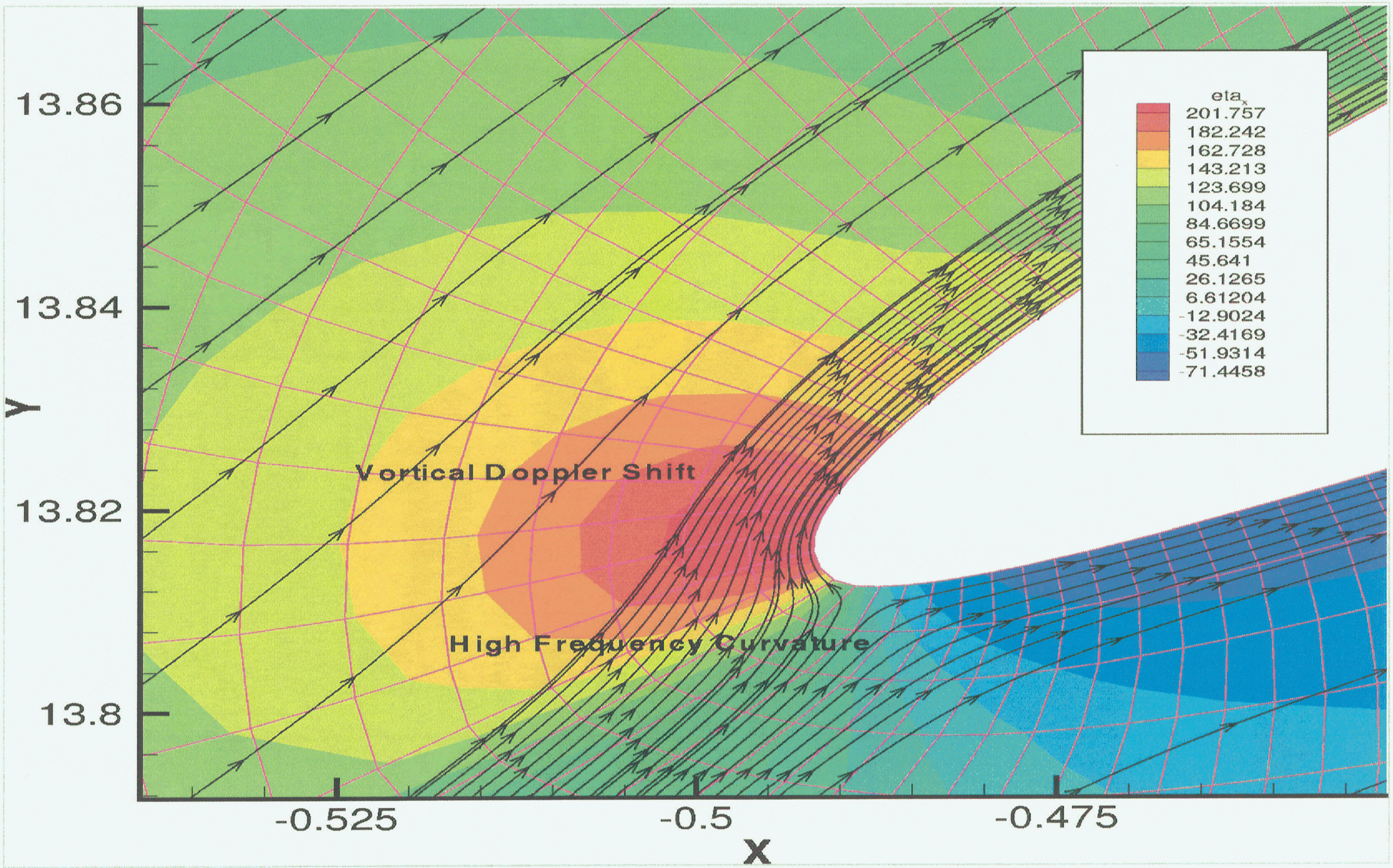


(a) (b)

Spectral Volume Grid

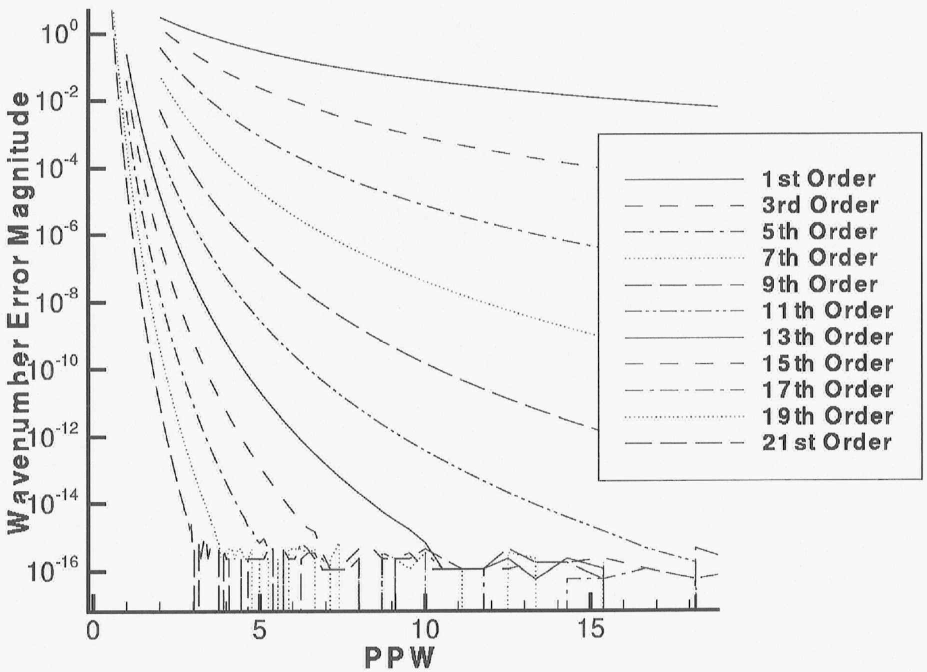
Equivalent Control Volume Grid

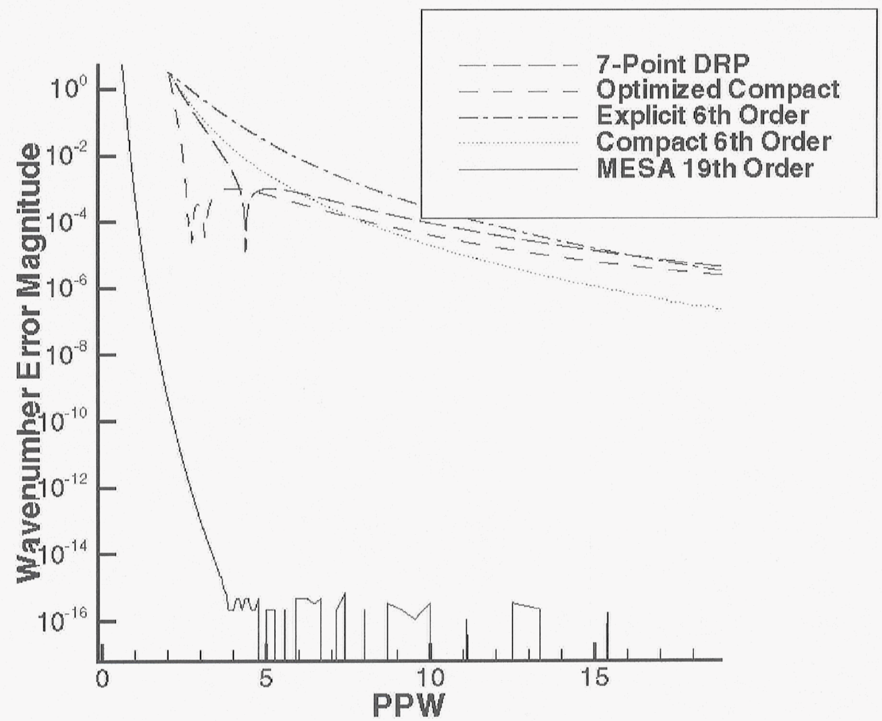
Higher Order Curvilinear Surfaces



Computing very high order derivatives of metrics is new

Higher Order Derivatives Improves Spatial Resolution





Ultra High Fidelity Hermitian Integration, Flux, and Interpolation

GRC
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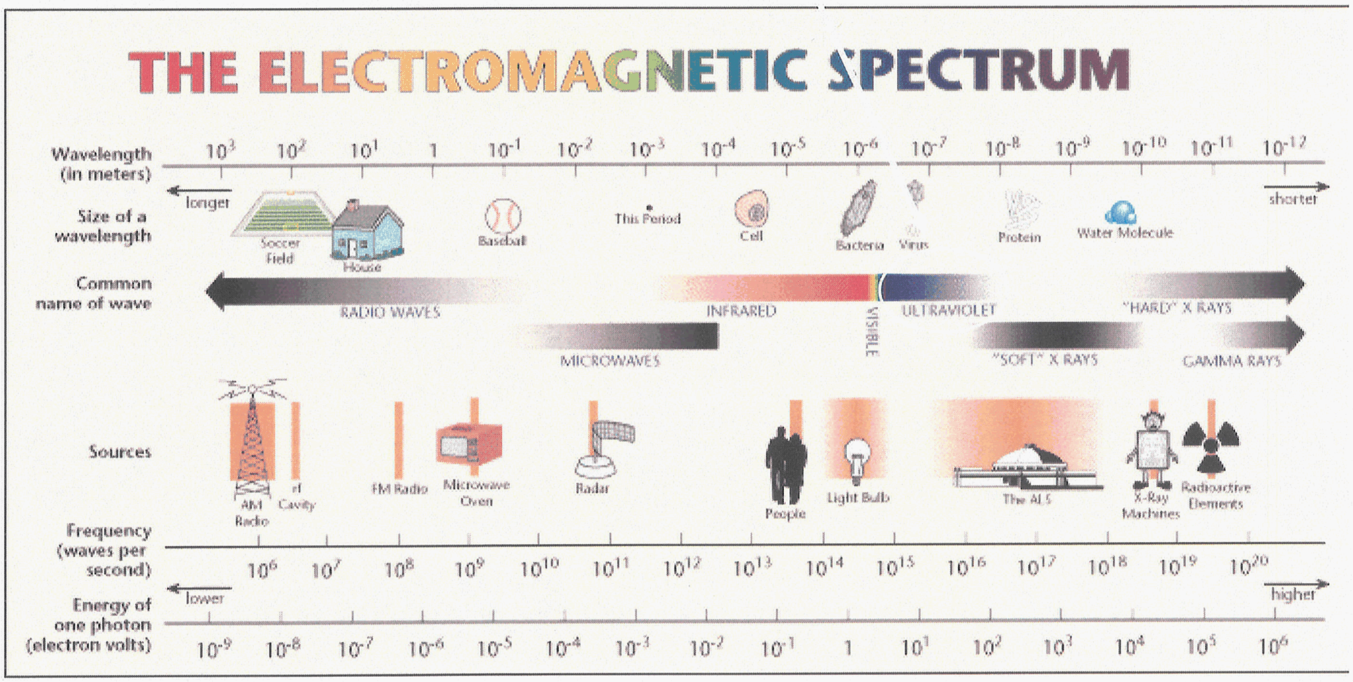
- Resolves in the “ultra” wavelength range
  - Ultra-short waves (less than two mesh spacings, CKW Tam, 2002) or ultra wavelength scaling (10e3 thru 10e-9)



Advancing in Time

- Shorthand Notation for Derivatives

$$\overset{\chi}{C}{}^{f}_{a,b,k} = \frac{\partial^{a+b+k} f(x,y,t)}{\partial x^a y^b t^k}$$

- All time accurate methods effectively do this:

$$p(x,y,t+\Delta t) = C^p_{0,0,0} + \boxed{C^p_{0,0,1}}\,\Delta t + \boxed{C^p_{0,0,2}}\,\frac{\Delta t^2}{2!} + \ldots$$

Time Derivatives as Space Derivatives—1st Order

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$$-C^p_{0,0,1} = C^u_{0,0,0}C^p_{1,0,0} + C^v_{0,0,0}C^p_{0,1,0} + \gamma C^p_{0,0,0}\left(C^u_{1,0,0} + C^v_{0,1,0}\right)$$

Must correct p_x, p_y, u_x, v_y at surface to attain 1st order time accuracy

Time Derivatives as Space Derivatives—2nd Order

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CORRECTING

p_x, p_y, p_{xy}, p_{xx}, p_{yy}

u_x, u_y, u_{xy}, u_{xx}

v_x, v_y, v_{xy}, v_{yy}

ρ_x, ρ_y

1st Order Correction

2nd Order Correction

1st Order Entropy

$$
\begin{aligned}
-C^p_{0,0,2} = \\
& C^p_{1,0,0}\left(C^u_{0,0,0}C^u_{1,0,0} + C^v_{0,0,0}C^u_{0,1,0} + C^{1/\rho}_{0,0,0}C^p_{1,0,0}\right) + \\
& C^u_{0,0,0}\left(C^u_{1,0,0}C^p_{1,0,0} + C^u_{0,0,0}C^p_{2,0,0} + C^v_{1,0,0}C^p_{0,1,0} + \right. \\
& C^v_{0,0,0}C^p_{1,1,0} + \gamma C^p_{1,0,0}\left(C^u_{1,0,0} + C^v_{0,1,0}\right) + \\
& \left.\gamma C^p_{0,0,0}\left(C^u_{2,0,0} + C^v_{1,1,0}\right)\right) + \\
& C^p_{0,1,0}\left(C^u_{0,0,0}C^v_{1,0,0} + C^v_{0,0,0}C^v_{0,1,0} + C^{1/\rho}_{0,0,0}C^p_{0,1,0}\right) + \\
& C^v_{0,0,0}\left(C^u_{0,1,0}C^p_{1,0,0} + C^u_{0,0,0}C^p_{1,1,0} + C^v_{0,1,0}C^p_{0,1,0} + \right. \\
& C^v_{0,0,0}C^p_{0,2,0} + \gamma C^p_{0,1,0}\left(C^u_{1,0,0} + C^v_{0,1,0}\right) + \\
& \left.\gamma C^p_{0,0,0}\left(C^u_{1,1,0} + C^v_{0,2,0}\right)\right) + \\
& \gamma\left(C^u_{1,0,0} + C^v_{0,1,0}\right)\left(C^u_{0,0,0}C^p_{1,0,0} + C^v_{0,0,0}C^p_{0,1,0} + \right. \\
& \left.\gamma C^p_{0,0,0}\left(C^v_{0,0,0}C^p_{0,1,0}\right)\right) + \\
& \gamma C^p_{0,0,0}\left(-\left(C^u_{1,0,0}C^u_{1,0,0} + C^u_{0,0,0}C^u_{2,0,0} + C^v_{1,0,0}C^u_{0,1,0} + \right.\right. \\
& \left.\left.C^v_{0,0,0}C^u_{1,1,0} + C^{1/\rho}_{1,0,0}C^p_{1,0,0} + C^{1/\rho}_{0,0,0}C^p_{2,0,0} + \right)\right) + \\
& \gamma C^p_{0,0,0}\left(-\left(C^u_{0,1,0}C^v_{1,0,0} + C^u_{0,0,0}C^v_{1,1,0} + C^v_{0,1,0}C^v_{0,1,0} + \right.\right. \\
& \left.\left.C^v_{0,0,0}C^v_{0,2,0} + C^{1/\rho}_{0,1,0}C^p_{0,1,0} + C^{1/\rho}_{0,0,0}C^p_{0,2,0}\right)\right)
\end{aligned}
$$

High Accuracy at Surfaces

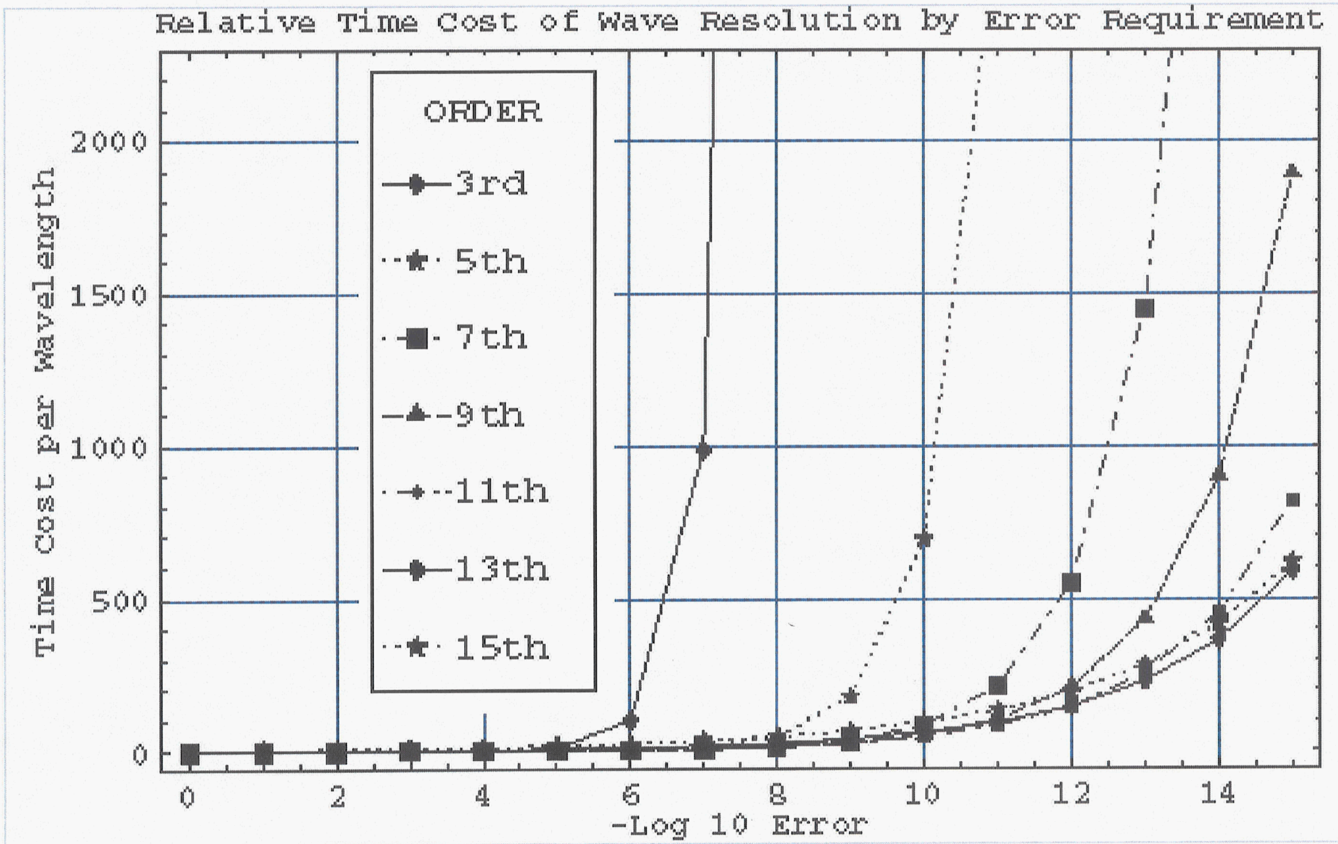
- Inviscid Surface Boundary Conditions:

$$\boxed{\frac{\partial^\alpha (\vec{V} \cdot \hat{n})}{\partial t^\alpha} = \frac{\partial^\alpha Q_5}{\partial t^\alpha} \forall \alpha : (\alpha \geq 0)}$$

- **Normally, only α=0 is used in CFD**

But to achieve higher order in time at a surface must use larger α (Goodrich, 1999)

Efficiency Chart



Dyson, J.Comp. Acoust, Vol. 10, No.2,2002

Need 6th order for 5 wavelengths (10^{-6})

Non-equilibrium Model (Gedeon)

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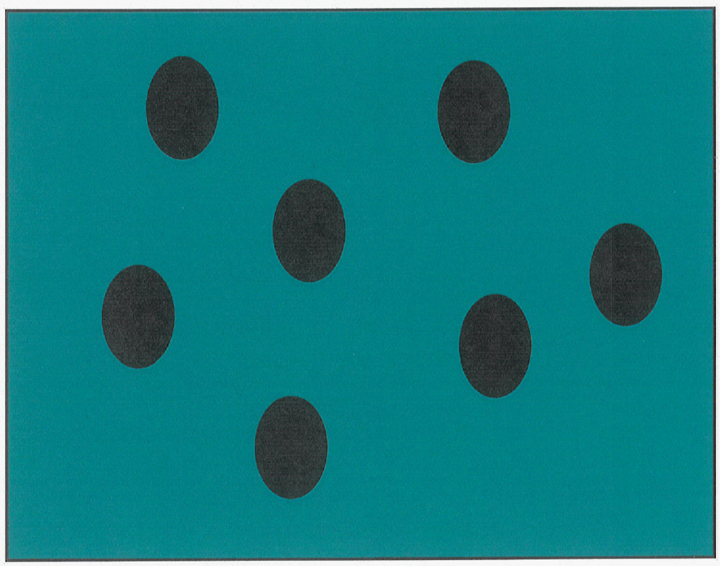
$$M = \rho \qquad \frac{\partial M}{\partial \tau} + \frac{J}{\beta}\frac{\partial}{\partial y_k}\left(\frac{G^k}{J}\right) = 0$$

$$G = \beta\rho V \qquad \frac{\partial G^n}{\partial \tau} + \left(J\frac{\partial}{\partial y_k}\left(\frac{G^k}{J}u_j - \frac{\beta}{J}\tau_{eij}\frac{\partial y_k}{\partial x_i}\right) + \beta J \frac{\partial}{\partial y_k}\left(\frac{P}{J}\frac{\partial y_k}{\partial x_j}\right) + \beta\phi_{jk}u_k \right)\frac{\partial y_n}{\partial x_j} = 0$$

$$E = \rho e \qquad \frac{\partial E}{\partial \tau} + \frac{J}{\beta}\frac{\partial}{\partial y_k}\left(\frac{E}{J}\beta u^k + \frac{1}{J}(P\beta V)^k - \frac{1}{J}(\tau_e \cdot \beta V)^k + \frac{\beta}{J}q_e^k\right) - Q = 0$$

$$\frac{\partial}{\partial \tau}(\lambda T_s) + Q = 0$$



- Void-Average Velocity Assumption may not be good in first 20 layers due to density gradients
- Curvilinear formulation enables better surface heat transfer

Conclusions/Suggestions

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- Use high order parallelizable methods
- Use DES for oscillating transition modeling
- Use parallel technology (Myrinet, Infiniband, Octa-Opteron)

- Need ultra short wave resolution
- Easily parallelizable
- Multiple Time-Stepping
- Use High order boundary conditions
- High fidelity in space and time
- Geometry definition naturally has high fidelity
- Look at Spectral element methods, Discontinous Galerkin, etc.

- Extend SAGE/GLIMPS to 3D
- Extend Manifest to parallel and/or use latest CFD
- Validate Manifest model with DNS
- Use regularized regenerator geometry
- Perform DNS in first 20 layers where non-uniform and model the rest

- Numerical Error (Dissipation/Dispersion)
- Modeling Error- Sliding Interfaces,Faceted Geometry, Turbulence Transition